

Semihard diffractive production of neutral mesons by off shell photons and the range of pQCD validity*

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Abstract

We study the dependence on photon virtuality Q^2 for the semihard quasi-elastic photoproduction of neutral vector mesons on a quark, gluon or real photon (at $s \gg p_\perp^2$, Q^2 ; $p_\perp^2 \gg \mu^2 \approx (0.3 \text{ GeV})^2$). To this end we calculate the corresponding amplitudes (in an analytical form) in the lowest nontrivial approximation of the perturbative QCD. The amplitudes for the production of mesons consisting of light quarks vary very rapidly with the photon virtuality near $Q^2 = 0$.

We estimate the bound of the pQCD validity region for such processes. For the process with mass shell photon the obtained bound is very high, and this region seems beyond opportunities of real experiment. This bound decreases fast with the increase of Q^2 , and we expect that the virtual photoproduction at HERA provides the opportunity to test the pQCD results. The signature of this region is discussed.

1 Introduction

The diffractive photoproduction of neutral vector mesons V is studied in many theoretical [1–12] and experimental [13–15] papers.

In this paper we study this photoproduction on quark, gluon or other photon, initiated by off shell photon γ^* (with virtuality Q^2):

$$\gamma^* q \rightarrow Vq, \quad \gamma^* g \rightarrow Vg; \quad \gamma^* \gamma \rightarrow VV'. \quad (1)$$

in the region of parameters where the perturbative QCD (pQCD) validity is beyond doubts:

$$s \gg p_\perp^2, \quad Q^2; \quad p_\perp^2 \gg \mu^2 \quad (\mu \approx 0.2 \div 0.3 \text{ GeV}). \quad (2)$$

The transverse momentum of produced meson relative to collision axis p_\perp is small as compared to energy but it is large as compared to QCD scale $\mu \approx 0.3 \text{ GeV}$.

These reactions can be studied in the photoproduction on proton with the rapidity gap $\eta > \eta_0$ between produced meson and other produced hadrons X . *We denote such processes as diffractive photoproduction.* The cross section of the process $\gamma^* p \rightarrow VX$ with

*The earlier version of this paper was distributed as preprint [1]

rapidity gap is related with that for the photoproduction on quark and gluon via the well known relation¹:

$$\frac{d\sigma(\gamma^*p \rightarrow VX)}{dtdx} = \sum_f (q(x,t) + \bar{q}(x,t)) \frac{d\sigma(\gamma^*q \rightarrow Vq)}{dt} + G(x,t) \frac{d\sigma(\gamma^*G \rightarrow VG)}{dt}; \quad x > \frac{4p_\perp^2}{s} \cosh^2 \frac{\eta_0}{2}.$$

The photon–photon collisions of the discussed type can be also studied at the future photon colliders [16].

To estimate the bounds of the pQCD validity region, we simulate the nonperturbative effects near these bounds by the specific model. Its idea is to use the pQCD equations, in which quark mass is considered as a parameter (which is near the constituent quark mass). We also use this model for the qualitative description of some phenomena below this bound.

Our efforts are focused on the problems: *What are main features of Q^2 -dependence in these processes within pQCD, without any phenomenological hypotheses? What are the bounds for pQCD validity at the description of diffractive processes?* To this end we restrict our consideration by calculation in the lowest nontrivial approximation of pQCD — two–gluon exchange for production of vector mesons (Fig. 1). The obtained results provide opportunity to discuss the relation between the point–like and hadron–like components of a photon in the discussed reactions as well.

The known for us papers, treated similar problems, are discussed briefly in the last section.

The study of pQCD validity in the discussed processes is on line with that in other exclusive processes. The relation between perturbative and nonperturbative contributions in their description and the bounds for pQCD validity are discussed widely (see refs. [17, 18] and references therein). The advantage of processes considered is the possibility to study this subject by two probes simultaneously — via investigation of dependence on both the produced meson transverse momentum p_\perp and the photon virtuality Q^2 .

2 Basic relations

The process discussed can be described as two stage one. At the first stage, photon fragments into a $q\bar{q}$ pair, the quarks with energies ε_i move along the photon momentum. Their transverse momenta are relatively small and the total energy of quark’s pair is close to the energy of the photon, $\varepsilon_1 + \varepsilon_2 \approx E$. This first stage describes also the processes with production of jet-like system (both resolved for two quark jets and unresolved one) with rapidity gap.

At the second stage then quarks are glued into meson.

The basic kinematical notations are presented in Fig. 1. We denote also the virtuality of photon by $Q^2 \equiv -p_1^2 > 0$, the quark mass — by m , the transverse momentum of produced meson (relative to the collision axis) — by \mathbf{p}_\perp . For the description of the photon fragmentation into quarks, we use quark spinors $u_1 = u(q_1)$ and $u_2 = u(-q_2)$. The relative motion of the quark and antiquark is described by a variable ξ that is the ratio:

¹ $G(x,t)$ and $q(x,t)$ are the gluon and quark densities in proton.

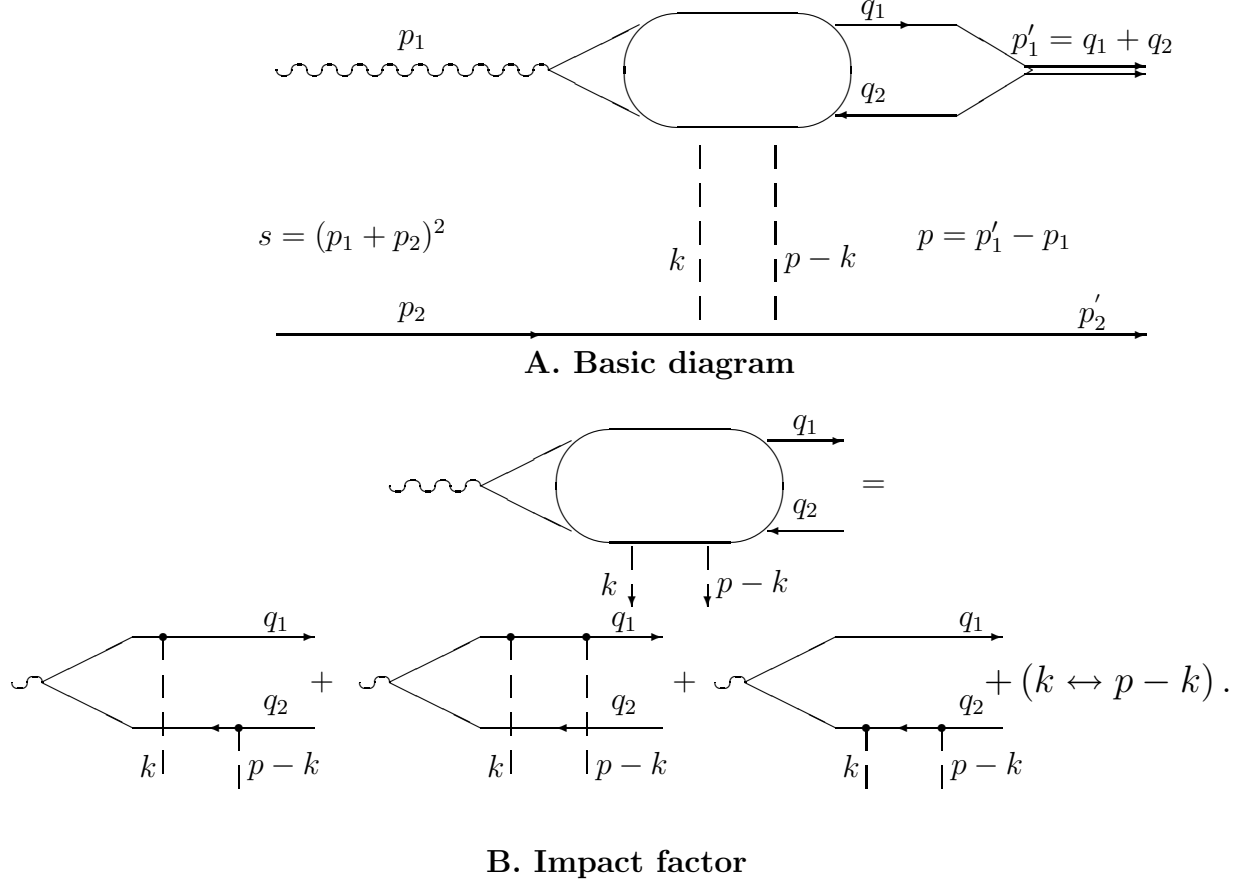


Figure 1: *Photoproduction of vector meson on a quark (two-gluon exchange).*

$$\xi = \frac{2(q_1 - q_2)p_2}{s} \equiv \frac{\varepsilon_1 - \varepsilon_2}{E}; \quad -1 \leq \xi \leq 1. \quad (3)$$

Next, we denote

$$\mathbf{n} = \frac{\mathbf{p}_\perp}{|\mathbf{p}_\perp|}; \quad \delta = \frac{2m}{p_\perp}; \quad u = \frac{Q^2}{\mathbf{p}_\perp^2}; \quad v = \delta^2 + (1 - \xi^2)u = \frac{4m^2 + (1 - \xi^2)Q^2}{\mathbf{p}_\perp^2}. \quad (4)$$

Besides, $e = (0, \mathbf{e}, 0)$ and $e_V = (0, \mathbf{e}_V, 0)$ are the polarization vectors of the transverse photon and the transversely polarized vector meson (in the state with helicity $\lambda = \pm 1$).

As usually, $\alpha_s = g^2/4\pi$, $\alpha = e^2/4\pi = 1/137$, $Q_q e$ is the quark charge, $N = 3$ is the number of colors.

Impact representation

The amplitude of the process in the lowest nontrivial order of pQCD is described by diagrams of Fig. 1 (with accuracy $\sim p_\perp^2/s$, Q^2/s). Just as in refs. [7, 8, 19], the sum of these diagrams is transformed with the same accuracy to an integral over the gluon

transverse momentum — *the impact representation*:

$$M_{\gamma^* q \rightarrow V q} = is \int \frac{J_{\gamma^* V}(\mathbf{k}_\perp, \mathbf{p}_\perp) J_{qq}(-\mathbf{k}_\perp, -\mathbf{p}_\perp)}{\mathbf{k}_\perp^2 (\mathbf{k}_\perp - \mathbf{p}_\perp)^2} \frac{d^2 k_\perp}{(2\pi)^2}. \quad (5)$$

Impact factors $J_{\gamma^* V}$ and J_{qq} correspond to the upper and lower blocks in Fig. 1. They are s -independent. The entire dependence on the photon virtuality is concentrated in the impact factor $J_{\gamma^* V}$. For colorless exchange impact factors include factors δ_{ab} , where a and b are the color indices of the exchanged gluons.

The impact factors for the transition between two colorless states vanish when the gluon momenta tend to zero [7]. This general property takes place independent on validity of perturbation theory. (In the coordinate space this property can be treated as zero's color charge of object. This property is named "dipole shielding", "quark coherence", etc.). In our case this property is written as

$$J_{\gamma^* V}(\mathbf{k}_\perp, \mathbf{p}_\perp) \rightarrow 0 \quad \text{at} \quad \begin{cases} \mathbf{k}_\perp \rightarrow 0, \\ (\mathbf{p} - \mathbf{k})_\perp \rightarrow 0. \end{cases} \quad (6)$$

The derivation of impact representation and impact factors repeats in main features that given in Appendices to ref. [7] (see refs. [20, 21] also) with two variations.

First, Sudakov variables are introduced precisely for the reaction with "massive" collided particles. All momenta are decomposed over "almost light-like" vectors combined from initial ones: $p'_1 = p_1 - (p_1^2/s)p_2 = p_1 + (Q^2/s)p_2$, $p'_2 = p_2 - (p_2^2/s)p_1$, and in the plane perpendicular to them. Simple calculations with these vectors show that the masses squared in the denominators of quark propagators become more "heavy":

$$m^2 \rightarrow m^2 + Q^2(1 - \xi^2)/4. \quad (7)$$

Second, the impact factors for these photons are different for the production by transverse (γ_T^*) and scalar (or longitudinal) (γ_S^*) off shell photons.

When consider the entire pQCD series in the leading log approximation (LLA), one can use the method of calculation from ref. [23]. In this case the impact representation transforms to the form (see Fig. 2):

$$M_{\gamma^* q \rightarrow V q} = is \int J_{\gamma^* V}(\mathbf{k}_\perp, \mathbf{p}_\perp) J_{qq}(-\mathbf{k}'_\perp, -\mathbf{p}_\perp) \mathcal{P}(s; p_\perp, \mathbf{k}_\perp, \mathbf{k}'_\perp) \frac{d^2 k_\perp d^2 k'_\perp}{(2\pi)^4}. \quad (8)$$

The discussed lowest nontrivial approximation of pQCD (5) corresponds to

$$\mathcal{P}(p_\perp, \mathbf{k}_\perp, \mathbf{k}'_\perp) = \frac{(2\pi)^2 \delta(\mathbf{k}_\perp - \mathbf{k}'_\perp)}{\mathbf{k}_\perp^2 (\mathbf{k}_\perp - \mathbf{p}_\perp)^2} \delta_{aa'} \delta_{bb'}.$$

The kernel \mathcal{P} of this equation relates to the perturbative Pomeron (pP). The impact factors, obtained in the basic approximation of pQCD, are also valid for the description of process in LLA for both mass shell and off shell photons until virtuality Q^2 is not too large.

In its asymptotic form the amplitude (8) is Regge-like:

$$M = is G_{\gamma^* V}(p_\perp, Q^2) \cdot K(s/p_\perp^2) \cdot G_{qq}. \quad (9)$$

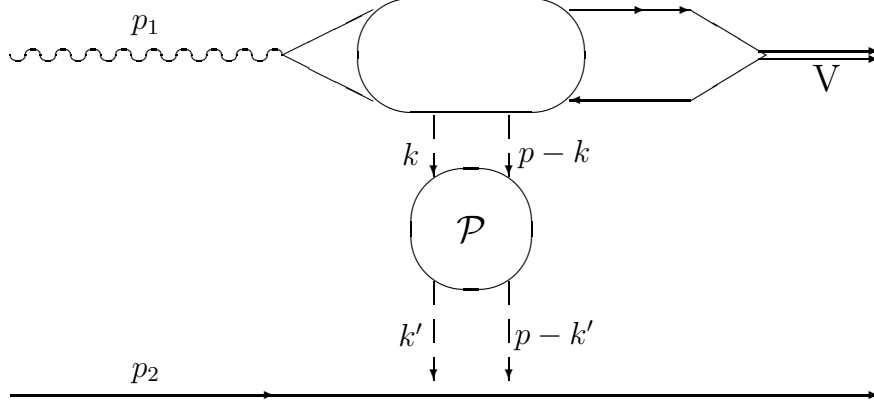


Figure 2:

The kernel K is pP itself, it is obtained in refs. [22]. Each vertex G_{γ^*V} is the convolution of our γ^*V impact factor with some standard factor from \mathcal{P} . The corresponding integration is similar to that in our case, and the Q^2 dependence near mass shell is roughly the same. The detailed calculation of Q^2 dependence here is absent now. The calculations of ref. [11] shows that this Regge-like form is valid at large enough $\eta \approx \ln(s/p_\perp^2) \gtrsim 3$ for real photons. At smaller values of rapidity gap η the lowest order calculations related to approximation of Fig. 1 seems more adequate for the description of data.

The theoretical and experimental study of pP is of great interest, since this object should be common for different reactions and it is sensitive to the inner structure of pQCD. In particular, it is important to test BFKL [22] predictions about pP in its pure form without mixing with large distance (soft) effects.

Quark and gluon impact factors

The impact factor for the colorless transitions $q \rightarrow q$ (from [7]) and $g \rightarrow g$ are similar:

$$J_{qq} = g^2 \frac{\delta_{ab}}{2N}; \quad J_{gg} = -g^2 \delta_{ab} \frac{N}{N^2 - 1}. \quad (10)$$

The helicity and color state of the quark or gluon target are conserved in these vertices.

Gluon dominance. The relations (10) shows that the cross section for the photoproduction of vector meson on a gluon is about 5 times larger than that on a quark:

$$d\sigma_{\gamma^*g \rightarrow Vg} = \left(\frac{2N^2}{N^2 - 1} \right)^2 d\sigma_{\gamma^*q \rightarrow Vq} = \frac{81}{16} d\sigma_{\gamma^*q \rightarrow Vq}. \quad (11)$$

It means that the photoproduction of vector meson on proton with a rapidity gap can be used for study of the gluon content of a proton.

Having in mind these facts, we will present the formulae for the photoproduction on the quark for the definiteness.

Impact factor $J_{\gamma^* q\bar{q}}$

The impact factor for a transverse photon has the same form as for the on shell photon [7] but with the replacement (7) in denominators:

$$J_{\gamma_T^* q\bar{q}} = eQ_q g^2 \frac{\delta_{ab}}{2N} \bar{u}_1 \left[mR(m)\hat{e} - (1+\xi)\mathbf{P}(m)\mathbf{e} - \hat{P}(m)\hat{e} \right] \frac{\hat{p}_2}{s} u_2. \quad (12)$$

Here transverse vector $P(m) = (0, \mathbf{P}(m), 0)$ and scalar $R(m)$ are:

$$\mathbf{P}(m) = \left[\frac{\mathbf{q}_{1\perp}}{\mathbf{q}_{1\perp}^2 + m^2 + (1-\xi^2)Q^2/4} + \frac{\mathbf{k}_\perp - \mathbf{q}_{1\perp}}{(\mathbf{k}_\perp - \mathbf{q}_{1\perp})^2 + m^2 + (1-\xi^2)Q^2/4} \right] - [\mathbf{q}_{1\perp} \leftrightarrow \mathbf{q}_{2\perp}]; \quad (13)$$

$$R(m) = \left[\frac{1}{\mathbf{q}_{1\perp}^2 + m^2 + (1-\xi^2)Q^2/4} - \frac{1}{(\mathbf{k}_\perp - \mathbf{q}_{1\perp})^2 + m^2 + (1-\xi^2)Q^2/4} \right] + [\mathbf{q}_{1\perp} \leftrightarrow \mathbf{q}_{2\perp}]. \quad (14)$$

To describe the impact factor for a scalar photon, it is necessary to know the polarization vector of scalar photon e_S . Taking into account the gauge invariance, one can use a reduced form of this vector $e_S = 2\sqrt{Q^2} (p_2/s)$ in our kinematical region. Then the calculations similar to those for T photon result in:

$$J_{\gamma_S^* q\bar{q}} = -eQ_q g^2 \frac{\delta_{ab}}{2N} \frac{1-\xi^2}{2} \sqrt{Q^2} R(m) \bar{u}_1 \frac{\hat{p}_2}{s} u_2. \quad (15)$$

It is easily seen that these impact factors obey eq. (6).

Impact factors for a meson production

To produce a meson, the relative transverse momenta of quarks should be small ($\lesssim \mu$). With our accuracy ($\mu^2/p_\perp^2 \ll 1$) the transverse momenta of quarks relative to the collision axis are proportional to their energies ε_i , i.e.

$$\mathbf{q}_{1\perp} = \frac{1}{2}(1+\xi)\mathbf{p}_\perp, \quad \mathbf{q}_{2\perp} = \frac{1}{2}(1-\xi)\mathbf{p}_\perp, \quad \varepsilon_{1,2} = \frac{1}{2}(1\pm\xi)E.$$

The $q\bar{q} \rightarrow V$ transition is described, as usual (see [24]), by change of product $\bar{u}_1 \dots u_2$ for the meson wave function $\varphi_V(\xi)$:

$$Q_q \bar{u}_1 \dots u_2 \rightarrow \frac{Q_V}{4N} \int_{-1}^1 d\xi \begin{cases} f_V^L \varphi_V^L(\xi) \text{Tr}(\dots \hat{p}_3) & \text{for } V_L \\ f_V^T \varphi_V^T(\xi) \text{Tr}(\dots \hat{e}_V^* \hat{p}_3) & \text{for } V_T. \end{cases} \quad (16)$$

(The trace over vector and color indices is assumed). The quantity Q_V relates to the quark charges in the meson V . The specific forms for these wave functions is given in eq. (26), eq. (31). We use the coupling constants from refs. [25, 26]:

	ρ^0	ω	ϕ	Ψ	Ψ'	Υ	Υ'	Υ''
$f_V, \text{ GeV}$	0.21	0.21	0.23	0.38	0.28	0.66	0.49	0.42
Q_V	$1/\sqrt{2}$	$1/(3\sqrt{2})$	$1/3$	$2/3$	$2/3$	$1/3$	$1/3$	$1/3$

The impact factors J_{γ^*V} are obtained by substitution of eq. (16) into eqs. (12),(15):

a) For a *transverse photon* we have two opportunities:

$$J_{\gamma_T^*V}(\mathbf{k}_\perp, \mathbf{p}_\perp) = \frac{1}{2}eQ_V g^2 \frac{\delta_{ab}}{2N} \int_{-1}^1 d\xi \left\{ \begin{array}{ll} (-f_V^L) \varphi_V^L(\xi) \xi (\mathbf{P}\mathbf{e}) & \text{for } V_L \\ f_V^T \varphi_V^T(\xi) mR (\mathbf{e}\mathbf{e}_V^*) & \text{for } V_T. \end{array} \right. \quad (17)$$

b) A *scalar photon* produces a longitudinal vector meson only:

$$J_{\gamma_S^*V_L}(\mathbf{k}_\perp, \mathbf{p}_\perp) = -\frac{1}{2}eQ_V g^2 \frac{\delta_{ab}}{2N} \int_{-1}^1 d\xi f_V^L \varphi_V^L(\xi) \frac{1-\xi^2}{2} \sqrt{Q^2} R. \quad (18)$$

Below we neglect difference between φ^L and φ^T , f^L and f^T .

It is useful to introduce dimensionless vector \mathbf{r} via equation $\mathbf{k}_\perp = (\mathbf{r} + \mathbf{n})p_\perp/2$. Then the above impact factors acquire the forms:

$$J_{\gamma^*V} = eQ_V g^2 \frac{\delta_{ab}}{2N} \frac{f_V}{|p_\perp|} \left\{ \begin{array}{ll} (\mathbf{e}\mathbf{F}_{T \rightarrow V_L}) & \text{for T-photon} \rightarrow \text{meson } V_L \\ \delta (\mathbf{e}\mathbf{e}_V^*) F_{T \rightarrow V_T} & \text{for T-photon} \rightarrow \text{meson } V_T \\ F_{S \rightarrow V_L} & \text{for S-photon} \rightarrow \text{meson } V_L. \end{array} \right. \quad (19)$$

$$\mathbf{F}_{T \rightarrow V_L} = - \int_{-1}^1 d\xi \varphi_V(\xi) \cdot \xi \left\{ \left[\frac{(1+\xi)\mathbf{n}}{v+(1+\xi)^2} + \frac{\mathbf{r} - \mathbf{n}\xi}{v+(\mathbf{r} - \mathbf{n}\xi)^2} \right] - [\xi \leftrightarrow -\xi] \right\}; \quad (20)$$

$$F_{T \rightarrow V_T} = \int_{-1}^1 \varphi_V(\xi) d\xi \cdot \mathcal{R}; \quad F_{S \rightarrow V_L} = - \int_{-1}^1 d\xi \sqrt{u}(1-\xi^2) \varphi_V(\xi) \cdot \mathcal{R} \quad (21)$$

$$\mathcal{R} = \left[\frac{1}{v+(1+\xi)^2} - \frac{1}{v+(\mathbf{r} + \mathbf{n}\xi)^2} \right] + [\xi \leftrightarrow -\xi].$$

3 The neutral vector meson photoproduction on a quark or gluon

To calculate amplitudes under interest we substitute these impact factors into eq. (5). The result for the meson production on a quark is

$$M_{\gamma^*q \rightarrow V_q} = i \frac{eQ_V g^4}{\pi} \frac{s f_V}{|p_\perp|^3} \frac{N^2 - 1}{N^2} \left\{ \begin{array}{ll} (\mathbf{e}\mathbf{n}) I_{T \rightarrow V_L} & \text{for T-photon} \rightarrow \text{meson } V_L \\ (\mathbf{e}\mathbf{e}_V^*) \delta \cdot I_{T \rightarrow V_T} & \text{for T-photon} \rightarrow \text{meson } V_T \\ I_{S \rightarrow V_L} & \text{for S-photon} \rightarrow \text{meson } V_L \end{array} \right. \quad (22)$$

with

$$I_a = \frac{1}{4\pi} \int \frac{F_a(\mathbf{r}, \mathbf{n})}{(\mathbf{r} - \mathbf{n})^2 (\mathbf{r} + \mathbf{n})^2} d^2r \equiv \int_{-1}^1 d\xi \varphi_V(\xi) \Phi_a(\xi) \quad (a = T \rightarrow V_L, T \rightarrow V_T, S \rightarrow V_L). \quad (23)$$

(For $a = T \rightarrow V_L$ the quantity $(\mathbf{n}\mathbf{F}_{T \rightarrow V_L})$ is used.)

Just as in refs. [7, 8], we integrated over component of vector \mathbf{r} along \mathbf{n} using residues. Last integration is trivial (but bulky). Then the quantities in eq. (22) get the form

$$\Phi_{T \rightarrow V_L} = \frac{\xi}{4(1-\xi^2-v)} \left[\frac{(1+\xi)^2-v}{(1+\xi)^2+v} \ln \frac{(1+\xi)^2+v}{2\sqrt{v}} - \frac{(1-\xi)^2-v}{(1-\xi)^2+v} \ln \frac{(1-\xi)^2+v}{2\sqrt{v}} \right]$$

$$\begin{aligned}
\Phi_{T \rightarrow V_T} &= \frac{1}{2(1 - \xi^2 - v)} \left[\frac{(1 + \xi)}{(1 + \xi)^2 + v} \ln \frac{(1 + \xi)^2 + v}{2\sqrt{v}} + \frac{(1 - \xi)}{(1 - \xi)^2 + v} \ln \frac{(1 - \xi)^2 + v}{2\sqrt{v}} \right] \\
\Phi_{S \rightarrow V_L} &= -\sqrt{u}(1 - \xi^2)\Phi_{T \rightarrow V_T} .
\end{aligned} \tag{24}$$

We will discuss below the scale of Q^2 -dependence for cross sections. Let us define this scale Λ^2 by equation

$$I_T(\Lambda^2) = \frac{1}{2}I_T(Q^2 = 0). \tag{25}$$

3.1 Production of mesons consisting of heavy quarks

The calculations and results below differ for the production of mesons consisting of heavy or light quarks. We begin with a more simple case of mesons consisting of heavy quarks (J/Ψ or Υ). It seems more clean since the large quark mass suppresses nonperturbative effects. The results are similar in main features to those obtained in ref.[8] for the mass shell photons. We will speak below about the J/Ψ meson photoproduction for definiteness.

For the wave function of discussed mesons we use the usual main approximation:

$$\varphi(\xi) = \delta(\xi). \tag{26}$$

With this wave function the impact factor $T \rightarrow V_L$ (for production of longitudinally polarized vector meson by transverse photon) vanishes. The deviation from the simple form (26) can be described by the quantity $\langle \xi^2 \rangle = \int \xi^2 \varphi(\xi) d\xi \sim 0.1$.

Let us begin with the **photoproduction on a quark**. The main results for the transverse photon coincide with those for real photons [8] with the replacement $\delta^2 \rightarrow \nu$. Finally, in eq. (22)

$$I_{T \rightarrow \Psi_T} = \frac{1}{2(\nu^2 - 1)}L(\nu); \quad L(\nu) = \ln \frac{(1 + \nu)^2}{4\nu}; \quad I_{S \rightarrow \Psi_L} = -\sqrt{u}I_{T \rightarrow \Psi_T}; \tag{27}$$

$$I_{T \rightarrow \Psi_L} = \frac{\langle \xi^2 \rangle}{(1 + \nu)^2} \left[1 - \frac{\nu}{\nu - 1}L(\nu) \right]; \quad \nu = (4m^2 + Q^2)/\mathbf{p}_\perp^2. \tag{28}$$

Therefore, the helicity conserves in these reactions. *The transverse photon produces mainly transverse Ψ .* The admixture of longitudinally polarized Ψ is $\sim (\langle \xi^2 \rangle)^2 \sim 0.01$ [8]. *Scalar photon produce longitudinal Ψ_L only.* The ratio of amplitude with production of transverse J/Ψ by T -photons to that with production of longitudinal J/Ψ by S -photons is $\sqrt{4m_c^2/Q^2}$, it is independent on p_\perp . At $Q^2 > 4m^2$ the dominant polarization becomes longitudinal.

The largest amplitudes $\gamma_T^* q \rightarrow \Psi_T q$ and $\gamma_S^* q \rightarrow \Psi_L q$ vanish at $p_\perp^2 = 4m_c^2 + Q^2$ (or $\nu = 1$). These zeroes shift strongly due to $\langle \xi^2 \rangle$ corrections. At the higher values of p_\perp these cross sections are small (cf. ref. [8]).

The shape of both main amplitudes is determined by the single function $I_{T \rightarrow \Psi_T}$. One can see that the scale Λ^2 of Q^2 dependence increases from the natural value $4m_c^2/2$ at small p_\perp to $\sim p_\perp^2/10$ at large enough p_\perp .

The similar calculations give us the amplitudes for the **production of two mesons V' , V consisting of heavy quarks in $\gamma\gamma$ collision**. We consider the production of both identical and different mesons by real or virtual photons. In particular, the collision of the virtual photon with the real one is described by two

nonzero amplitudes, the first — for the production by T -photon and the second — for the production by S -photon (these amplitudes are finite at $p_\perp \rightarrow 0$):

$$\begin{aligned}
M_{\gamma_T^* \gamma \rightarrow V'_T V_T} &= \frac{is}{p_\perp^4} \frac{e^2 g^4}{\pi} Q_V Q_{V'} f_V f_{V'} \frac{N^2 - 1}{N^2} (\mathbf{e}_1 \mathbf{e}_{V'}^*) (\mathbf{e}_2 \mathbf{e}_V^*) \delta \delta' \cdot I_{V'V}; \\
M_{\gamma_S^* \gamma \rightarrow V'_L V_T} &= -\frac{is}{p_\perp^4} \frac{e^2 g^4}{\pi} Q_V Q_{V'} f_V f_{V'} \frac{N^2 - 1}{N^2} (\mathbf{e}_2 \mathbf{e}_V^*) \sqrt{u'} \delta \cdot I_{V'V}; \\
I_{V'V} &= \frac{1}{u'} \left[\frac{L(\delta^2)}{(\nu' + 1)(\delta - 1)} - \frac{L(\nu')}{(\nu' - 1)(\delta + 1)} \right].
\end{aligned} \tag{29}$$

Here ν' corresponds to meson produced by off shell photon, and δ — to on shell one.

3.2 Production of mesons consisting of light quarks on a quark or gluon

The impact factor $J_{\gamma_T^* V_T}$ contains factor $\delta = 2m/p_\perp$. Therefore, **in the range of pQCD validity (at large enough p_\perp) the transverse photons produce mesons consisting of light quarks in the states with helicity 0 only** for any polarization of an initial photon². It is due to the chiral nature of perturbative couplings in the massless limit.

We write in this section for shortness I_T instead of $I_{T \rightarrow V_L}$ and

$$I_S \equiv I_{S \rightarrow V_L}(u) \equiv -\frac{2\sqrt{u}}{1+u}(I_T + U). \tag{30}$$

We use the wave functions of mesons consisting of light quarks in the form [25]:

$$\varphi_V(\xi) = \frac{3}{4} (1 - \xi^2) \left(1 - \frac{1}{5} b_V + b_V \xi^2 \right). \tag{31}$$

Coefficient b_V tends to 0 slowly with growth of p_\perp^2 ($b_\rho = b_\omega = 1.5$; $b_\phi = 0$ at $p_\perp \approx 1$ GeV).

For the asymptotical wave function ($b_V = 0$) we obtain:

$$\begin{aligned}
I_T(u) &\equiv I_0(u) = \frac{3}{8(1-u)^3} \left[2 + 10u - u \left(\frac{1+u}{1-u} \right) \left(\ln^2 \frac{1}{u} + 6 \ln \frac{1}{u} \right) \right]; \\
U(u) &\equiv U_0(u) = \frac{3}{8(1-u)} \left(2 - \frac{1+u}{1-u} \ln \frac{1}{u} \right).
\end{aligned} \tag{32}$$

For the case $b_V \neq 0$ the more complicated expressions are obtained:

$$\begin{aligned}
I_T(u) &= I_0(u) \left[1 - \frac{b_V}{5} + b_V \left(\frac{1+u}{1-u} \right)^2 \right] + \\
&+ \frac{b_V}{12(1-u)^4} \left[-(3 - 10u)(1-u) + 16u(u^2 + 5u + 1)U_0(u) \right]; \\
U(u) &= U_0(u) + \frac{b_V}{60(1-u)^2} \left[5(1-u) + 8(1 + 8u + u^2)U_0(u) \right].
\end{aligned} \tag{33}$$

² It is in contrast with well known fact, that at small p_\perp helicity conserves mainly, i.e. vector mesons are transversely polarized.

These expressions are regular at $u = 1$. At $u = 0$ (real photoproduction) $I_T = \frac{3}{4}(1 + \frac{7}{15}b_V)$ [7].

The dependence on photon virtuality is concentrated in factors I_T , I_S . The shapes of these functions depend weakly on the form of wave function (value of quantity b_V). They are plotted in Fig. 3 for the ρ^0 meson production ($b_V = 1.5$).

If the virtuality of photon is less than p_\perp^2 (or $u < 1$), the amplitude for transverse photon dominates over longitudinal one. Fig. 3 shows very sharp peak in I_T near $Q^2 = 0$. It is due to items $\propto u \ln^2 u$ in eqs. (32), (33). The derivative of amplitude in Q^2 (in u) diverges at $Q^2 = 0$, it is infrared unstable in contrast with amplitude itself, which is infrared stable. The quantity I_T is reduced by half at $u \approx 0.1$. It means, that the scale of Q^2 dependence here is

$$\Lambda_{pert}^2 \approx p_\perp^2 / 10. \quad (34)$$

The quantity $I_S(u)$ changes its sign at small enough $u = u_0$. $u_0 \approx 0.1$ for the asymptotical form of wave function ($b_V = 0$); and $u_0 \approx 0.02$ for the more wide wave function with $b_V = 1.5$. This behavior is similar to that for J/Ψ production.

If photon virtuality is large, $Q^2 > p_\perp^2$ (or $u > 1$), the amplitude with the scalar photon is dominant:

$$M_{\gamma_S^* q \rightarrow V_L q} \propto \frac{\ln u}{(Q^2)^{3/2}}; \quad M_{\gamma_T^* q \rightarrow V_L q} \propto \frac{p_\perp (\ln u)^2}{(Q^2)^2} \quad \left(u = \frac{Q^2}{p_\perp^2}\right). \quad (35)$$

The pQCD cross sections for the light vector meson production on gluon (the sum $d(\sigma_{\gamma_T^* g \rightarrow Vg} + d\sigma_{\gamma_S^* g \rightarrow Vg})/dp_\perp^2$) are presented in Fig. 3. They are s -independent in the used first nontrivial pQCD approximation.

Some remarks related to Q^2 dependence.

(i) The obtained scale of Q^2 dependence is substantially lower than it was expected before calculations. Using of this dependence will allow to improve the estimates for corresponding production rate at ep collisions.

(ii) Some features of Q^2 dependence for the high energy asymptotic of LLA result were obtained in ref. [11]. Here the picture at small Q^2 (near mass shell) is similar to that discussed above. Oppositely, far from mass shell (at $u \gg 1$) the LLA amplitude contains an additional factor $\propto Q/p_\perp$ in comparison with two-gluon approximation.

4 Small p_\perp limit for high Q^2 ($u < 1$) and coherence

In the pQCD limit the amplitudes of photoproduction on quark or gluon (27,32,33) diverge at $p_\perp \rightarrow 0$ (see [8] too). It means that "soft" nonperturbative region $k_\perp \lesssim \mu$ contributes substantially in this range and pQCD calculations are infrared unstable. Therefore, the details of p_\perp dependence at $p_\perp \rightarrow 0$ are out of range of the pQCD validity even at large Q^2 when we consider production on color target.

The reason for this divergence is simple: in the above limit the poles of both gluon propagators coincide, and we deal with the integral

$$\int J(\mathbf{k}_\perp) d^2 k_\perp / k_\perp^4.$$

In accordance with eq. (6), $J \propto \mathbf{k}_\perp$ at $\mathbf{k}_\perp \rightarrow 0$ and $J \propto (\mathbf{p} - \mathbf{k})_\perp$ at $(\mathbf{p} - \mathbf{k})_\perp \rightarrow 0$. At $\mathbf{p}_\perp \rightarrow 0$ both these zeroes coincide, and $J \propto k_\perp^2$. Then the discussed divergence is

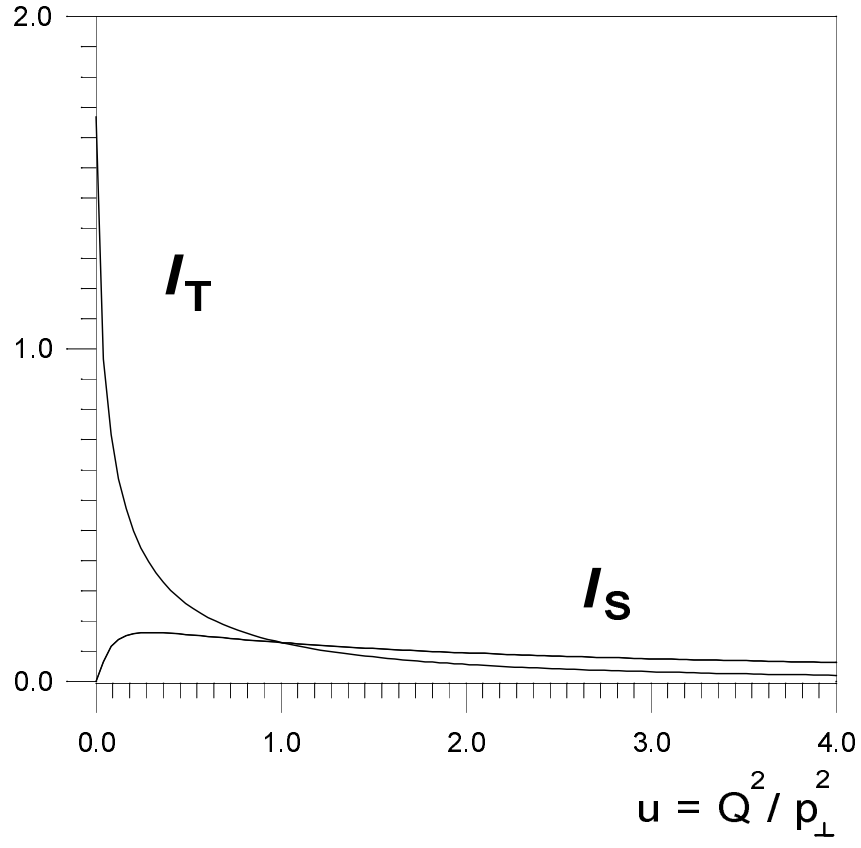


Figure 3: Functions I_T and I_S for the process $\gamma^* q \rightarrow \rho^0 q$ or $\gamma^* g \rightarrow \rho^0 g$.

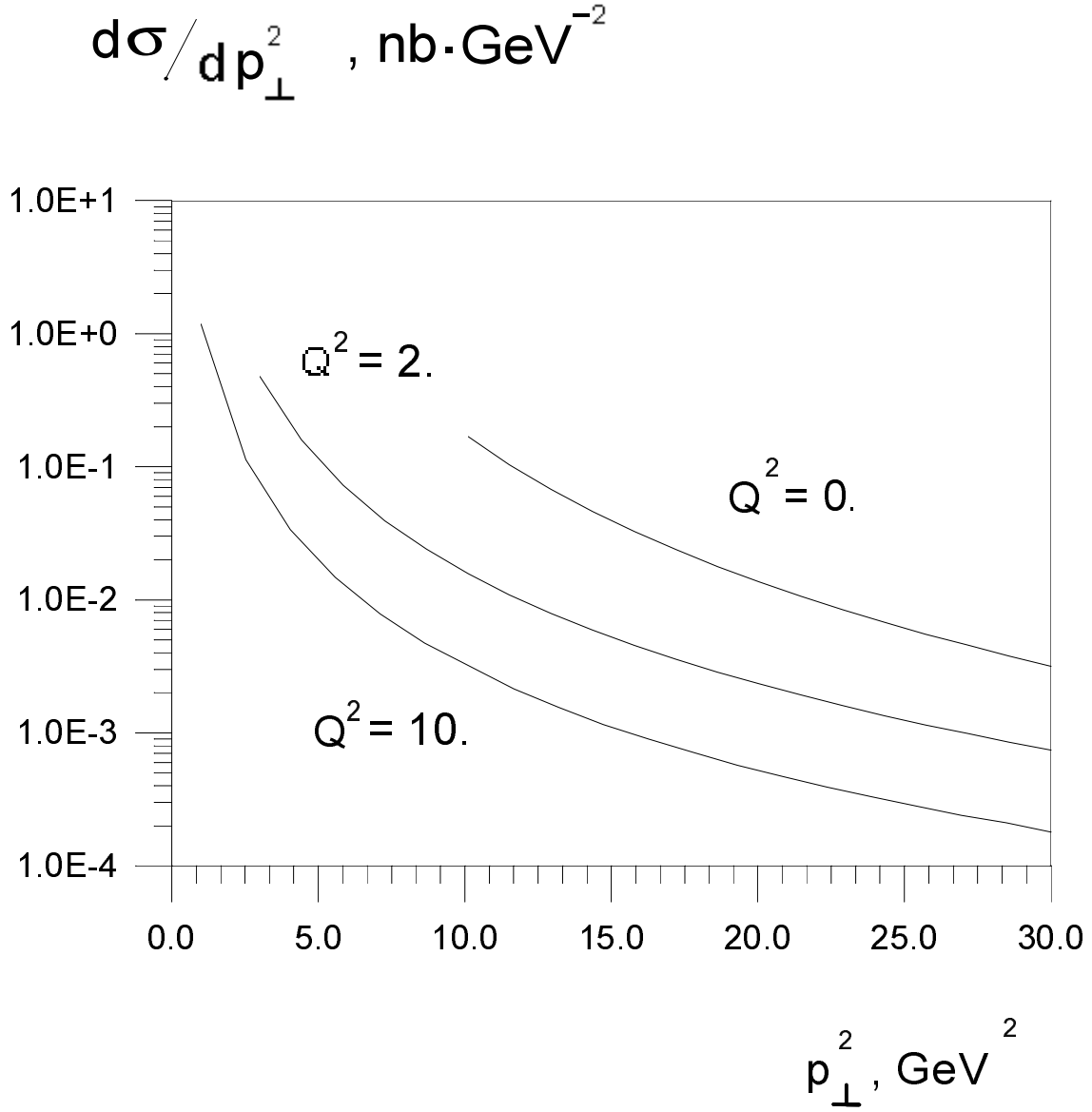


Figure 4: Differential cross section of $\gamma^*g \rightarrow \rho^0g$ process at $Q^2 = 0$, $Q^2 = 2$ GeV² and $Q^2 = 10$ GeV².

logarithmic one only and the above divergence is integrable, the total cross section is finite.

On the contrary, the $\gamma^*\gamma \rightarrow \Psi\Psi$ amplitude is finite at $p_\perp \rightarrow 0$ due to additional factor $J \propto k_\perp^2$ in the integrand. Therefore, soft part of integration region gives a negligible contribution here.

The main difference between these amplitudes originates from the fact that in the last case we deal with collision of two real colorless objects; coherence between quarks results in the additional suppression of soft nonperturbative contribution there (6).

The above comparison shows us that the coherence in both collided particles should be taken into account to describe phenomena at any p_\perp within pQCD even in the region of large Q^2 .

5 The range of validity of pQCD results

The above results show us that the using of pQCD for the description of experimental data can be inaccurate in some region of parameters. For example, the obtained scale of Q^2 dependence $\Lambda_{pert}^2 \approx p_\perp^2/10$ (34) is smaller than the natural scale of this dependence near mass shell $\Lambda_{soft}^2 \approx m_\rho^2$ even at $p_\perp = 2.5$ GeV when our small parameter $\mu^2/p_\perp^2 < 0.02$.

In this section we discuss the bounds of the pQCD validity region in dependence of p_\perp for *the photoproduction of vector mesons consisting of light quarks*, like ρ (provided $s \gg p_\perp^2$). Below we use some single scale of QCD nonperturbative effects (confinement, gluon correlations, etc.) — μ . We will have in mind the value $\mu = 0.2 \div 0.3$ GeV, which is close to the confinement scale, constituent quark mass, etc.

In the discussion below we assume the impact representation to be valid independent on validity of pQCD for description of different factors in it. In particular, the proof of impact representation in the lowest nontrivial approximation of pQCD is valid even in the regions near the poles of quark propagators in the impact factor, where its perturbative form (13,14) becomes incorrect.

The model for amplitude near the bound of region of pQCD validity.

To study the bound of pQCD validity, we simulate nonperturbative effects by adding of quantity μ^2 (instead of m^2) in all quark propagator denominators (assuming $\mu = 200 \div 300$ MeV)³. Besides, we change the quantity m from the quark propagator nominator (in front of item R in eq. (12)) for some new quantity $A \sim \mu$:

$$\mathbf{P}(m) \rightarrow \mathbf{P}(\mu); \quad mR(m) \rightarrow A \cdot R(\mu) \quad (A \sim \mu). \quad (36)$$

The regions, where the amplitude is sensitive to value of μ , are beyond pQCD validity. We denote the bound \mathbf{p}_{pert} of pQCD validity region by relation

$$d\sigma(p_\perp \geq p_{pert}|\mu) > 0.5 \cdot d\sigma(p_\perp \geq p_{pert}|\mu = 0). \quad (37)$$

We begin with *the meson photoproduction by real (on shell) photons*.

Contribution of item R provides helicity conservation (production of V_T) in the process. The item \mathbf{P} in the impact factor gives longitudinal polarization of produced mesons

³ The nonperturbative effects in the vicinity of poles of gluon propagators are suppressed due to property (6).

(V_L). The contribution of this item decreases more slow with p_\perp due to extra power of momentum in nominator. Therefore, this item gives amplitude at high enough p_\perp .

Let us discuss the limit $\mu \ll p_\perp$ in more detail. The contribution of R diverges in this limit due to integration near the poles of quark propagators at $k_\perp = q_{i\perp}$, it is $\sim \ln(p_\perp^2/\mu^2)$ (i.e. infrared unstable). It dominates at not too large p_\perp . Oppositely, the contribution of \mathbf{P} is finite in the discussed limit. It is infrared stable, and it defines amplitude within the range of pQCD validity.

Therefore, it is natural to assume that **the contribution \mathbf{P} describes the point-like component of photon** in the region where confinement effects are negligible. It dominates at high values of p_\perp and it provides production of longitudinally polarized mesons. Similarly, **the contribution \mathbf{R} for transverse photons describes the hadron-like component of photon**. It dominates at not too high values of p_\perp and it provides helicity conservation here. In addition to the boundary p_{pert} (37) we denote boundary value \mathbf{p}_{hel} by condition: At $p_\perp > p_{hel}$ the mean helicity of produced V changes from transversal to longitudinal one (i.e. hadron-like component R becomes relatively small).

The bound of pQCD validity region, estimate of p_{pert}

We expect that $p_{hel} < p_{pert}$. Therefore, to find the bound p_{pert} , one should consider point-like component of photon (contribution \mathbf{P} in impact factor) only. We present two estimate here.

First estimate. It is well known that the typical scale of the Q^2 dependence for soft processes $\Lambda_{soft}^2 \approx m_\rho^2$ (here m_ρ is the ρ meson mass). The known data shows us that this scale increases with p_\perp growth.

The scale of Q^2 dependence obtained is $\Lambda_{pert}^2 \approx p_\perp^2/10$ for the ρ photoproduction (34). The pQCD can be valid for description of the discussed phenomena if only $\Lambda_{pert}^2 > \Lambda_{soft}^2$, i.e. at $p_\perp^2/10 > m_\rho^2$ which leads to $p_\perp \gtrsim 3$ GeV. It does not contradict more refined estimate below (38).

Second estimate. We calculated numerically the contribution of item \mathbf{P} in impact factor (22)–(24) with some finite value of μ for different meson wave functions. Results — the ratios of $\Phi = M(\delta, u)/M(\delta = 0, u)$, — are shown in Fig. 5 for the important case of real photons ($u = 0$).

Naturally, the ratio $\Phi \rightarrow 1$ at $(p_\perp/\mu) \rightarrow \infty$. That is pQCD limit. We define value p_{pert} by condition $\Phi(p_{pert}, \mu) = 0.7$. At higher values of p_\perp influence of confinement effects for pQCD result in cross section is described by factor Φ^2 which is between 0.5 and 1.

It is seen that for mass shell photons $(p_{pert}/\mu) \approx 30 \div 40$. In this region the coefficient b_V in the ρ meson wave function decreases up to $b_V \approx 0.7$. Taking this fact into account, we have for $\mu = 0.2$ GeV

$$p_{pert} \approx \begin{cases} 7.5 \text{ GeV} & \text{for } b_V = 0.7 \quad (\rho - \text{meson at } p_\perp \approx 7 \text{ GeV}), \\ 6.2 \text{ GeV} & \text{for } b_V = 0 \quad (\phi - \text{meson}). \end{cases} \quad (38)$$

For $\mu = 0.3$ GeV these quantities should be 1.5 times larger.

The obtained values of p_{pert} (38) for the mass shell photons are very high. It is because the correction to the pQCD result is governed by the parameter $\mu^2/p_\perp^2 \ln^2(p_\perp^2/\mu^2)$ but not the "natural" parameter μ^2/p_\perp^2 . The effect of " μ corrections" in pQCD equations is

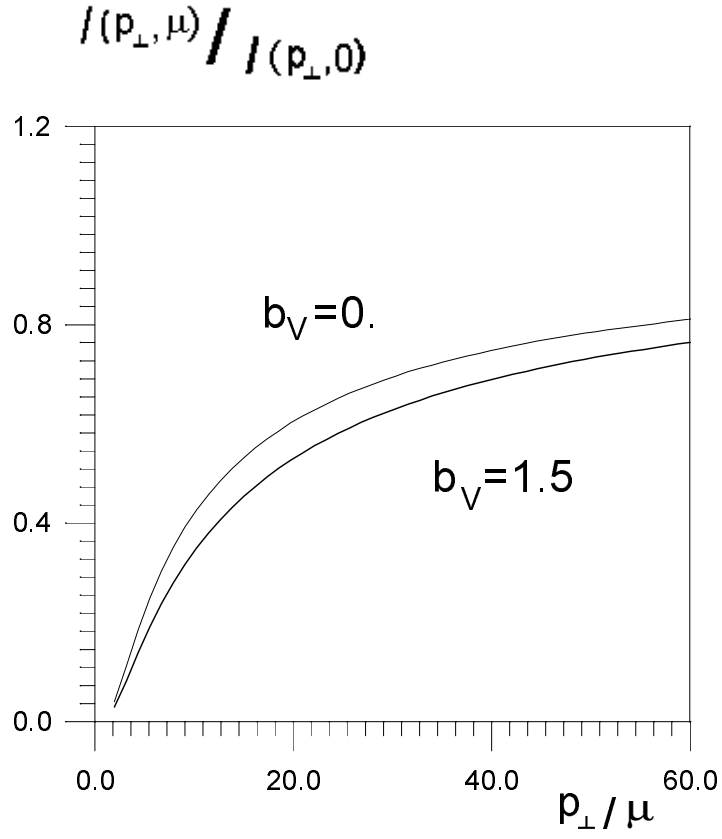


Figure 5: The ratios $\Phi = M(\delta)/M(\delta = 0)$ for mass shell photons ($u = 0$) in dependence on $p_{\perp} / \mu = 2\delta^{-1}$.

enhanced near the bounds of kinematical region, at $\xi \rightarrow \pm 1$. Therefore, their influence is higher for the wave function, which is "shifted" to these bounds (with $b_V > 0$). It corresponds to the table 1. In other words, the bounds for pQCD validity region p_{pert} are lower for the ϕ meson photoproduction ($b_V = 0$) in comparison with that for ρ photoproduction ($b_V = 1.5$). The photon virtuality prevents quark propagators from their poles while $\xi \neq \pm 1$. It is the reason why p_{pert} decreases fast with photon virtuality. For example, Our calculations show that

$$\begin{aligned}
p_{pert}^2 &\approx \left\{ \begin{array}{ll} 1.3 \text{ GeV}^2 & \text{for } \rho \\ 1 \text{ GeV}^2 & \text{for } \phi \end{array} \right\} \text{ at } \mu = 0.2 \text{ GeV}, Q^2 = 1 \text{ GeV}^2; \\
p_{pert}^2 &\approx \left\{ \begin{array}{ll} 28 \text{ GeV}^2 & \text{for } \rho \\ 10 \text{ GeV}^2 & \text{for } \phi \end{array} \right\} \text{ at } \mu = 0.3 \text{ GeV}, Q^2 = 1 \text{ GeV}^2; \\
p_{pert}^2 &\approx \left\{ \begin{array}{ll} 3.3 \text{ GeV}^2 & \text{for } \rho \\ 2 \text{ GeV}^2 & \text{for } \phi \end{array} \right\} \text{ at } \mu = 0.3 \text{ GeV}, Q^2 = 2.25 \text{ GeV}^2;
\end{aligned} \tag{39}$$

$$\tag{40}$$

It is seen, that the the pure pQCD description with longitudinal meson photoproduction become valid ealier in the ϕ photoproduction as compare with ρ one.

Signature of the pQCD validity for discussed processes. Polarization of produced mesons. Estimate of p_{hel}

The above description shows that the signature of pQCD validity is given by polarization of mesons consisting of light quarks and the correct dependence on p_\perp .

First, in the range of pQCD validity *these mesons should be produced in the state with helicity 0 only*. This result takes place for the production of both vector and tensor mesons [7, 19]. It is in strong contrast with the production in "soft" region where helicity conservation takes place, and real photons produce transversaly polarized mesons. (On the contrary, at the production of mesons consisting of heavy quarks the photon helicity transmits to meson in the main approximation.)

Second, the number of independent variables in the description of cross section is reduced from three to two⁴:

$$p_\perp^6 \frac{d\sigma}{dp_\perp^2} = F(\eta, u); \quad \eta = \ln(s/p_\perp^2), \quad u = \frac{Q^2}{p_\perp^2}. \tag{41}$$

Besides, the striking feature of results obtained is the very *sharp dependence on photon virtuality near $Q^2 = 0$* (more precise, on ratio u for reactions (1). The observation of such a behavior will be a good additional test of pQCD.

Next point is to see for the crossover point, in which the longitudinal polarization become dominant for the transversal initial photon (the boundary p_{hel}). This boundary is below boundary p_{pert} . Therefore, the calculations near this point depend on detail of model more strong. To see qualitative features of this crossover, the model (36) is used, in which we fix coefficient $A = 1 \text{ GeV}$ for definiteness.

⁴ This statement is valid for both discussed two-gluon approximation and LLA. In the last case the dependence of the size of rapidity gap η corresponds to the perturbative Pomeron.

Figs. 6,7 shows the cross sections of photoproduction by real photons for $\mu = 200$ MeV. In these figures curves R correspond to the production of transverse mesons (helicity conserved contribution, item R (14), hadron-like component of photon) and curves P correspond to the production of longitudinal mesons (item \mathbf{P} (13), point-like component of photon). Fig. 6 shows curves for the ρ^0 meson production ($b_V = 1.5$). Fig. 7 shows curves for the ϕ meson production (asymptotical wave function, $b_V = 0$).

In both cases the crossover point p_{hel} is $1.5 \div 5$ GeV. Next, the admixture of transversely polarized mesons at $p_\perp > p_{hel}$ for the ρ photoproduction decreases with p_\perp faster than that for ϕ mesons. We expect, that this feature conserves for virtual photons. It means, that in the data averaged over some p_\perp interval the fraction of longitudinal ϕ 's is larger than that for ρ 's. This conclusion is supported by data [14].

6 Brief discussion about some related papers

All known for us papers, which treat the similar problems, contain the essential phenomenological components (usually pQCD inspired).

These models are based, in fact, on the impact representation like (5). The description begins with the diffractive region (small p_\perp). It is one of reasons why authors use the hadron-like component of photon (item R but no point-like one \mathbf{P}) only with some parameter μ for detail description of cross section. Therefore, these models predict the helicity conservation in reaction $\gamma q \rightarrow \rho^0 q$. They don't predict change of polarization of produced mesons at high p_\perp .

The quasi-elastic process $\gamma^* p \rightarrow \rho p$ (without proton's dissociation) was studied in refs. [2, 3]. In these papers it was used the QCD inspired phenomenological model, which can be represented as impact representation (5) with the replacement of pQCD gluon propagators on the reggeized ones. The Q^2 dependence for the forward scattering in this model differs from that obtained in pQCD [5].

Recently the papers [5, 6, 4] were published, where the problems are studied that are close to those discussed above. In these papers quasi-elastic photoproduction of vector mesons on proton without proton's dissociation ($\gamma^* p \rightarrow V p$) is studied (with $V = \rho$ in [5, 4] and $V = J/\Psi$ in [6]).

The first stage in these papers corresponds to the simplest pQCD diagram just as in our paper. The following stages are used some features of processes at $p_\perp \approx 0$. To describe the picture at large p_\perp some phenomenological assumptions were added in papers [6, 4].

The ρ meson photoproduction at $p_\perp \approx 0$ was studied in ref. [5]. The same very region for the J/Ψ photoproduction is the starting point for ref. [6]. Just in this case some features of LLA provides an opportunity to use unitarity for construction of cross section in terms of the LLA proton's gluon distribution⁵. We consider quite other kinematical region (2).

The papers [4] treat the process (1). The crucial point here is using of hadron-like component of photon (factor R). It is the reason why these authors obtain transversal polarization of ρ meson for on shell photoproduction even at large enough p_\perp .

⁵ This basic construction is broken up at $p_\perp \neq 0$. To describe the J/Ψ production in this region, it is used the additional assumption in ref. [6] that the object, which was the proton's gluon distribution at $p_\perp = 0$, transforms to the product of this distribution and some proton form factor dependent on p_\perp^2 only.

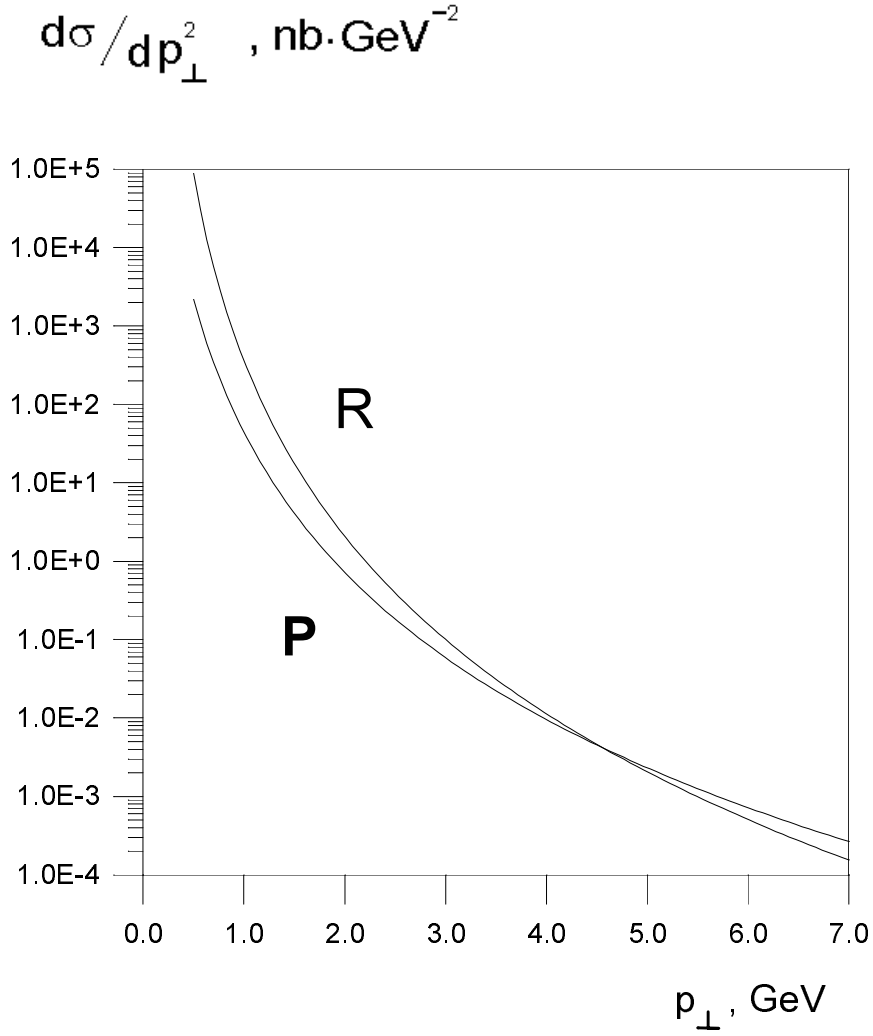


Figure 6: The diffrenential cross sections of ρ^0 meson photoproduction ($b_V = 1.5$) by real photons for $\mu = 200$ MeV. In these figures curves R correspond to the production of transverse mesons (helicity conserved contribution, item R , hadron-like component of photon) and curves P correspond to the production of longitudinal mesons (item P , point-like component of photon).

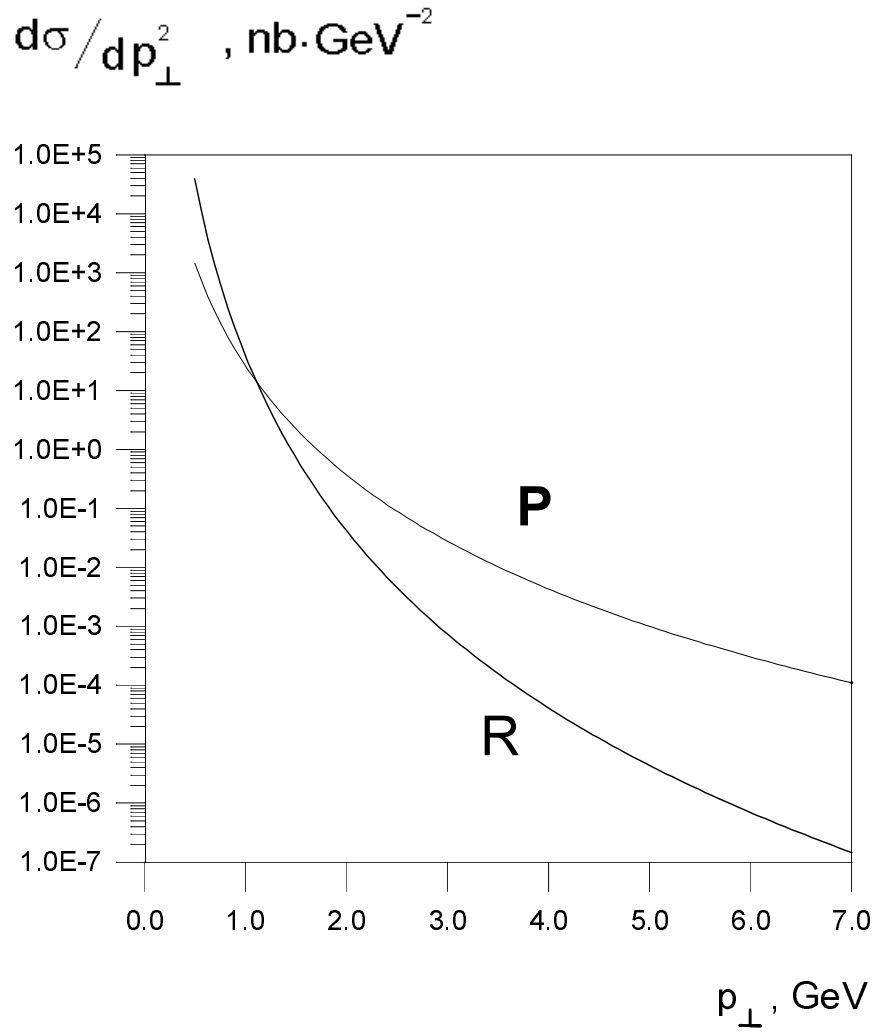


Figure 7: The same figure as previous one, but for photoproduction of ϕ meson ($b_V = 0.0$)

The photoproduction of pseudoscalar or tensor mesons — processes with three gluon exchange in the t channel (like $\gamma^* q \rightarrow \pi^0 q$) relates to the Odderon problem. For the mass shell photons these processes were considered in the similar approach in ref. [19].

The impact representation like (5) with three gluon denominators and impact factors similar to those in eq. (10) describe these processes [19]).

These processes with virtual photon were also calculated. Here the last integrations were numerical ones [28]. The results obtained are similar to those for discussed case. The new curve I_T is similar to that in Fig. 3, but they are more sharp at small u as compare with the 2-gluon exchange (Fig. 3).

Note that the photoproduction of scalar (or tensor) meson on a gluon is forbidden due to C parity conservation [27]. Therefore, the comparative study of the vector and scalar (or tensor) meson production in ep collision can give an additional information about a gluon content of proton and shadowing effects at small x .

7 Concluding remarks

Let us summarize our predictions (mainly for HERA experiments) related to the photoproduction of mesons consisting of light quarks.

1. For the real photoproduction we expect change of mean polarization of produced vector mesons at $p_\perp \sim 1.5 \div 5$ GeV. Above this bound produced vector mesons should be mainly longitudinally polarized. We expect that for the ϕ photoproduction this bound is lower than that for ρ , and the fraction of transversal ϕ decreases with p_\perp more fast.
2. The pure pQCD regim is hardly observable for real photons since the corresponding boundary values are very high, even for $\mu = 0.2$ GeV (38). This regim can be seen better in photoproduction by virtual photons (39). The signatures for this regim are:
 - Mesons are polarized longitudinally.
 - The number of independent variables is reduced up to two in the description of quantity $p_\perp^{-6}(d\sigma/dp_\perp^2)$ (41).
3. One can consider the special region of large enough p_\perp (within the region of pQCD validity) and not too high values of rapidity gap (say, $y < 3$). In accordance with the results of refs. [6, 11], we expect that in this region our two gluon approximation works good, i.e. the y -dependence is weak and u -dependence is given by eqs. (32,33).

The photoproduction of jets in the "direct" configuration and with the rapidity gap provides opportunity to see the same mechanisms in processes with larger cross sections. First data in this problem were reported recently [15]. The results of corresponding calculations are rather bulky and needs for detail discussions. One can expect that the point p_{pert} will be lower here than that for the vector meson production (38).

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